Writing mathematics exercise

One problem that we have with our students is that they cannot write mathematics. There are a number of reasons why we should change this. One is that it is a transferable skill (or whatever is this year's buzzword for this type of skill), i.e., the ability to make a coherent argument can be transfered to other areas, for example, in the world of paid work. Other reasons can be found on pages 3 to 4 of the *How to Write Mathematics* booklet:

`http://www.kevinhouston.net/pdf/htwm.pdf`.

Furthermore, there are videos on writing mathematics at

`http://www.youtube.com/user/DrKevinHouston`

One reason I think good writing is important is that a student forced to write clearly is forced to think clearly.

On page 3 of this file there is Exercise 3.2(ii) of the booklet. Feel free to copy this and distribute to students. It is a student's answer to a question about finding maxima and minima of functions. The exercise is to rewrite the answer so that it written well. A possible format to use is the following:

1. Get the students to read the solution.
2. Working individually or in pairs they should find any mistakes (mathematical or in the writing).
3. The individuals or pairs should then contribute their findings to the group and discuss. (This is a good opportunity to get students writing on the board.)
4. Either individually, in pairs, or even as a whole group, the students should rewrite the answer so that it is a perfect solution.

There are some mathematical mistakes. The second line is actually $f'(x)$ yet the student has equated it to f. Also they say (taking into account the previous comment) that $f'(x) = 6x^2 - 24x + 18$ implies that $x = 1$ and 3. This is not true, it is the *equation* $f'(x) = 0$ that gives us the values for x. Note also that rather than show that $f'(x) = 0 \iff x = 1$ or 3, the student shows $f'(x) = 0 \implies x = 1$ or 3. Equality signs are missing in the calculation of x. Some notation is missing: when finding maxima and minima we put the values of x into the expression for the second derivative and so we should really have

$$\left.\frac{d^2y}{dx^2}\right|_{x=1} \text{ or } \frac{d^2y}{dx^2}(1) \text{ rather than just } \frac{d^2y}{dx^2}.$$

Note also that in the sketch the axes are not labelled and there is no scale.

One possible solution to the exercise:

Let $f(x) = 2x^3 - 12x^2 + 18x$. Then $f'(x) = 6x^2 - 24x + 18$.
Hence

$$\begin{aligned} & f'(x) &=& \ 0 \\ \iff & 6x^2 - 24x + 18 &=& \ 0 \text{ [note the } \iff \text{, it's not } \implies] \\ \iff & x &=& \ \frac{24 \pm \sqrt{24^2 - 4 \times 18 \times 6}}{2 \times 6} \\ \iff & x &=& \ \frac{24 \pm \sqrt{144}}{12} \\ \iff & x &=& \ 2 \pm 1 \\ \iff & x &=& \ 1 \text{ or } 3. \end{aligned}$$

Since $\dfrac{d^2y}{dx^2} = 12x - 24$ we have

$$\left.\frac{d^2y}{dx^2}\right|_{x=1} = 12 \times 1 - 24 = -12 < 0$$

and hence $x = 1$ is a maximum. We have $f(1) = 2 - 12 + 18 = 8$.
Furthermore,

$$\left.\frac{d^2y}{dx^2}\right|_{x=3} = 12 \times 3 - 24 = 12 > 0$$

and hence $x = 3$ is a minimum. We have $f(3) = 2 \times 27 - 12 \times 9 + 18 \times 3 = 0$.

For the sketch students should include the x and $f(x)$ on the axes and indicate a scale on each axis.

Please get in touch if you have any comments, suggestions or anything else of interest!

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Writing mathematics

The following exercise is taken from *How to Think Like a Mathematician* by Kevin Houston (k.houston@leeds.ac.uk, www.kevinhouston.net).

Students were asked to 'Find the maximum and minimum values of the function $f(x) = 2x^3 - 12x^2 + 18x$ and sketch its graph.' One student submitted the following answer. Rewrite their answer so that it is written correctly.

$f = 2x^3 - 12x^2 + 18x$

$= 6x^2 - 24x + 18 \Rightarrow \quad x = \dfrac{24 \pm \sqrt{24^2 - 4 \times 18 \times 6}}{2 \times 6}$

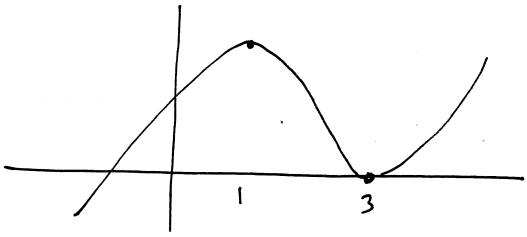
$\dfrac{24 \pm \sqrt{144}}{12}$

2 ± 1

$1, 3.$

$\dfrac{d^2y}{dx^2} = 12x - 24 \Rightarrow \dfrac{d^2y}{dx^2} = 12 \times 1 - 24 = -12 < 0$ max

$\dfrac{d^2y}{dx^2} = 12 \times 3 - 24 = 12 > 0$ min



$y = 2 - 12 + 18 = 8$

$y = 2 \times 27 - 12 \times 9 + 18 \times 3$

$= 0.$

Some points to consider:

- ❏ Write in sentences.
- ❏ Use punctuation.
- ❏ Don't start sentences with $\Longrightarrow$.
- ❏ The symbol $\Longrightarrow$ does not mean 'This is what we do next'.
- ❏ Explain what you are doing.

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