Complex Analysis May/June 2002.

1 (b) Determine the radii of convergence for the following series

(i) $\sum_{n=1}^{\infty}(3-4i)^n z^n$, (ii) $\sum_{n=1}^{\infty}\frac{i(2n)!}{n!}z^n$.

A

(b) (i) $\sum_{n=1}^{\infty}(3-4i)^n z^n$

$$\frac{|a_{n+1}|}{|a_n|} = \left|\frac{(3-4i)^{n+1}\, z^{n+1}}{(3-4i)^n\, z^n}\right| = (3-4i)|z|$$

$(3-4i)|z|$ cvgs if <1

$\therefore |z| < \frac{1}{3-4i}$

$\therefore$ Radius of cvgce, $R = \frac{1}{3-4i}$

(ii) $\sum_{n=1}^{\infty}\frac{i(2n)!}{n!}z^n$ $\qquad \frac{|a_{n+1}|}{|a_n|} = \left|\frac{i(2n+2)!\,(n+1)!\,z^{n+1}}{i(2n)!\;n!\;z^n}\right|$

$= (2n+1)(2n+2)(n+1)|z|$
$= (4n^2+4n+2n+2)(n+1)|z| = (4n^2+6n+2)(n+1)|z|$
$= (4n^3+10n^2+8n+2)|z|$

As $n\to\infty$, $\frac{|a_{n+1}|}{|a_n|}\to\infty$.

$\therefore R = 0$

B

b) i) $\sum_{n=1}^{\infty} (3-4i)^n z^n = \sum_{n=1}^{\infty} a_n z^n$

$a_n = (3-4i)^n z^n$

apply ratio test

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(3-4i)^{n+1}}{(3-4i)^n}\right|$$

$$= |(3-4i)|$$

this is independent of z and diverges as $n \to \infty$

so $R = \infty$

ii) $\sum_{n=1}^{\infty} \frac{i(2n)!}{n!} z^n$

$a_n = \frac{i(2n)!}{n!}$

ratio test: $\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\frac{i(2n+2)!}{(n+1)!}}{\frac{i(2n)!}{n!}}\right|$

$$= \left|\frac{i(2n+2)!\, n!}{(n+1)!\, i(2n)!}\right|$$

$$= \left|\frac{(2n+1)(2n+2)}{n+1}\right|$$

$$= \left|\frac{4n^2+7n+2}{n+1}\right|$$

as $n \to \infty$ $\left|\frac{a_{n+1}}{a_n}\right| \to \infty$ since $n^2 \to \infty$ so again $R = \infty$

(c)

b). radii of Convergence.

i) $\sum_{n=1}^{\infty} (3-4i)^n z^n \Rightarrow$ use ratio test.

$$\frac{\left|(3-4i)^{n+1} z^{n+1}\right|}{\left|(3-4i)^n z^n\right|} = (3-4i)z$$

radius of Convergence $= 0$.

ii) $\sum_{n=1}^{\infty} \frac{i(2n)!}{n!} z^n \Rightarrow$ use ratio test.

$$\frac{\frac{i(2(n+1))!\, z^{n+1}}{(n+1)!}}{i(2n)!/n!\, z.n}$$

$$= \frac{i(2n+2)!\, z^{n+1}}{(n+1)!} \times \frac{n!}{i(2n)!} \cdot z^n \Rightarrow \frac{\cancel{i}(2n+2)!\, n!}{(n+1)!\, \cancel{i}(2n)!} \cdot z^n \cdot z^{n+1}$$

$$\frac{(2n+1)(2n+2)n!\, z^{2n+1}}{(n+1)!} \quad \text{as } n \to \infty \quad z^{2n+1} \to \infty$$

the radius of Convergence is ∞

1.

(b) (i) $\sum_{n=1}^{\infty} (3-4i)^n z^n$ (D).

let $a_n = (3-4i)^n z^n$

& $\frac{a_{n+1}}{a_n} = \frac{(3-4i)^{n+1} z^{n+1}}{(3-4i)^n z^n} = (3-4i)z$

∴ z converges for $|z| < \frac{1}{(3-4i)}$

that is $R = \frac{1}{3-4i}$

(ii) $\sum_{n=1}^{\infty} \frac{i(2n)!}{n!} z^n$

Let $a_n = \frac{i(2n)!}{n!} z^n$

then $\frac{a_{n+1}}{a_n} = \frac{i(2(n+1))! z^{n+1}}{i(2n)! z^n} \frac{n!}{(n+1)!}$

$= \frac{(2n+2)!}{(2n)!} \frac{1}{(n+1)} \cdot z$

$= \frac{(2n+2)(2n+1)(2n)!}{(2n)!(n+1)} z$

$= \frac{(2n+2)(2n+1)}{(n+1)} z$

as $n \to \infty$, we see $\frac{(2n+2)(2n+1)z}{(n+1)} \to \infty$

$\forall |z|$. so the radius of convergence, $R = 0$.