Teaching Students to Write Mathematics: Leeds 12th May 2011

There's more to maths than 'doing the maths'

Mike Robinson
Sheffield Hallam Department of Engineering and Mathematics

Acknowledgements

Staff and student participants in the More Maths Grads HE Curriculum Theme, especially team members **Neil Challis** and **Mike Thomlinson.**

more maths grads
multiplying opportunities

Sheffield Hallam University

One member of staff…

“What really annoys me is that they use equals signs with gay abandon…”

Writing well is intricately linked to core skills

Clear thinking about the process

Logical arguments

Being precise

Producing meaningful graphs

Being accurate

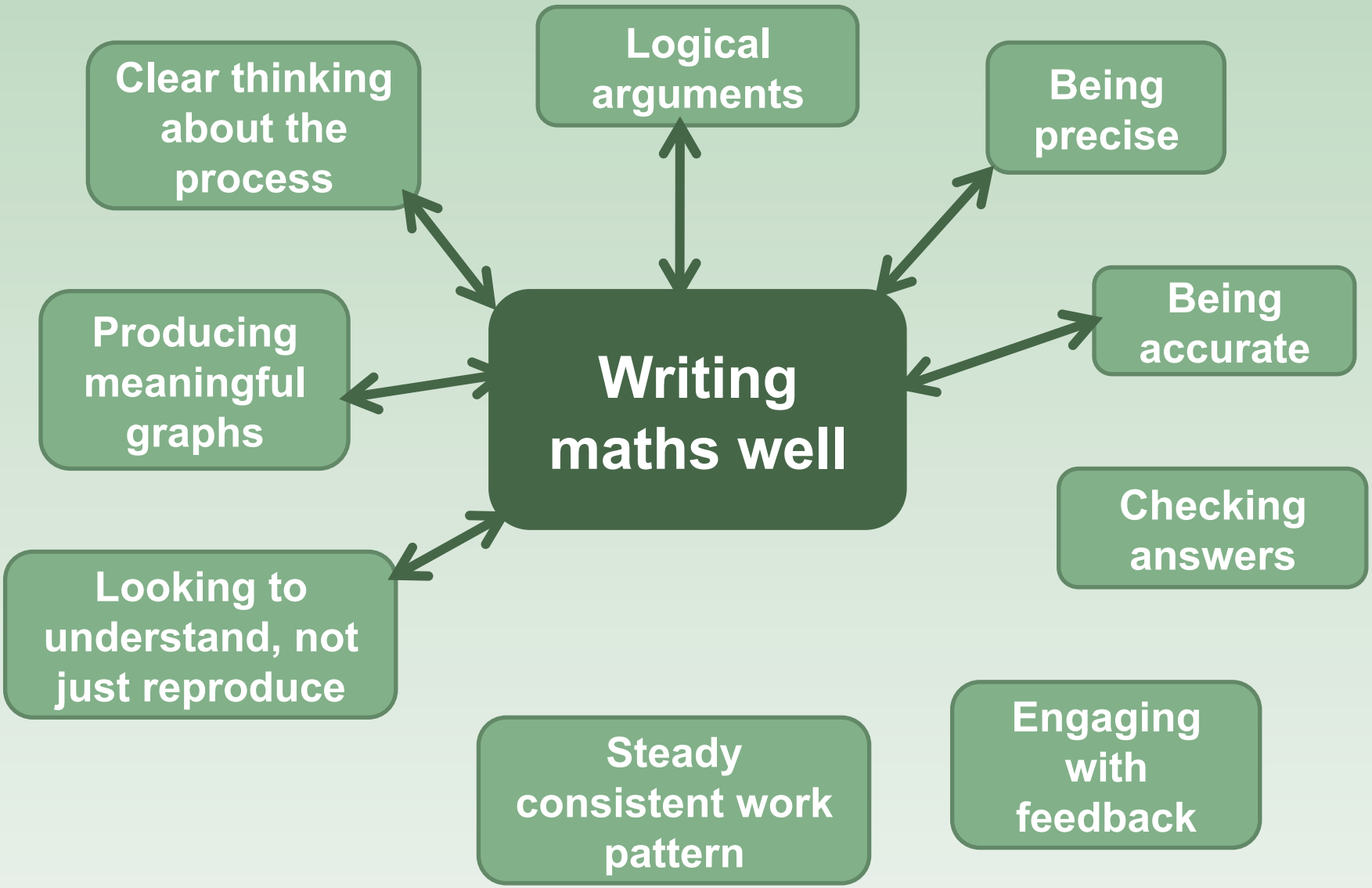
Checking answers

Looking to understand, not just reproduce

Steady consistent work pattern

Engaging with feedback

Writing well is intricately linked to core skills



One member of staff…

“What really annoys me is that they use equals signs with gay abandon…”

A fresh approach to Mathematical Methods

1. Changing my own attitude to teaching the core skills
   - it **is** my job to teach them this. *Stop moaning about it!*
2. Explicit emphasis on core, mathematical skills
3. Very gradual development of the skills
4. New emphasis reflected in all aspects of the course
   - introduction and course design
   - handouts
   - lectures / teaching
   - assessment / mark scheme...

Reflected in: *Introduction to the module*

- Module guide, first paragraph:

Introduction

The aims of this unit are:

- for you to develop the skills to read, think, work and write as a professional mathematician…

Reflected in: *Introduction to the module*

- Module guide, first paragraph:

Introduction

The aims of this unit are:

- for you to develop the skills to read, think, work and write as a professional mathematician…

- Week 1 Handout starts

  "Writing your maths well is important because…"

At which point, a minor diversion…

What is 'good writing'? Whose benefit?

I tell them that writing good maths is important because

- you make fewer mistakes
- it forces you into sound thinking
- maths should be precise, watertight, logical (…) and sloppily written work never satisfies this.

What is 'good writing'? Whose benefit?

I tell them that writing good maths is important because

- you make fewer mistakes
- it forces you into sound thinking
- maths should be precise, watertight, logical (…) and sloppily written work never satisfies this.

And yes, because…

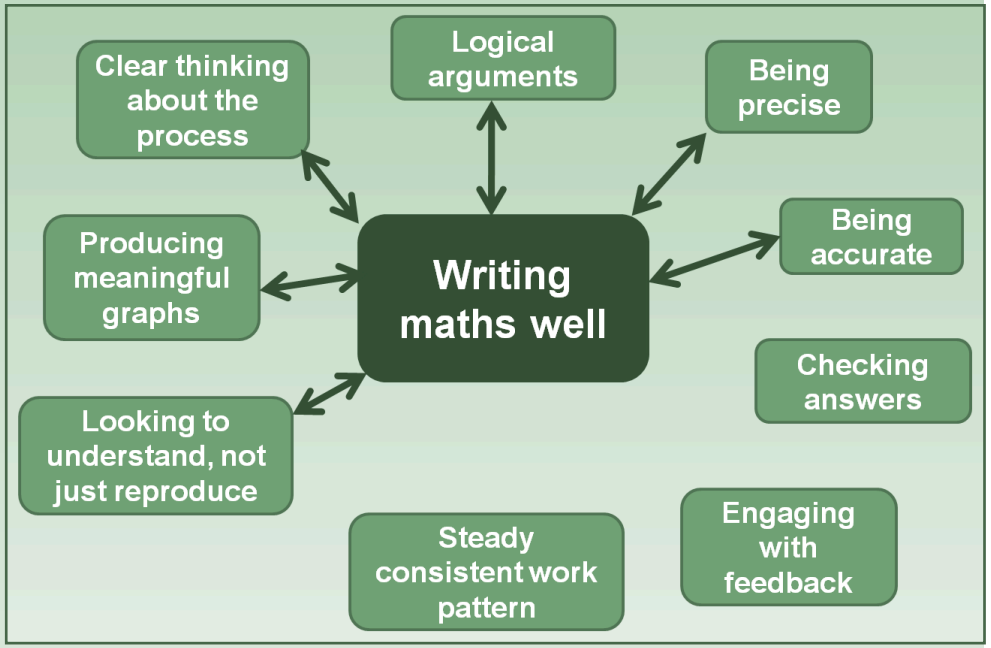
- being able to communicate your ideas to others important
- it's your responsibility to communicate clearly.

And even…

- pity the poor marker who has 90 scripts to get through, or
- remember that the marker is the one awarding the marks.

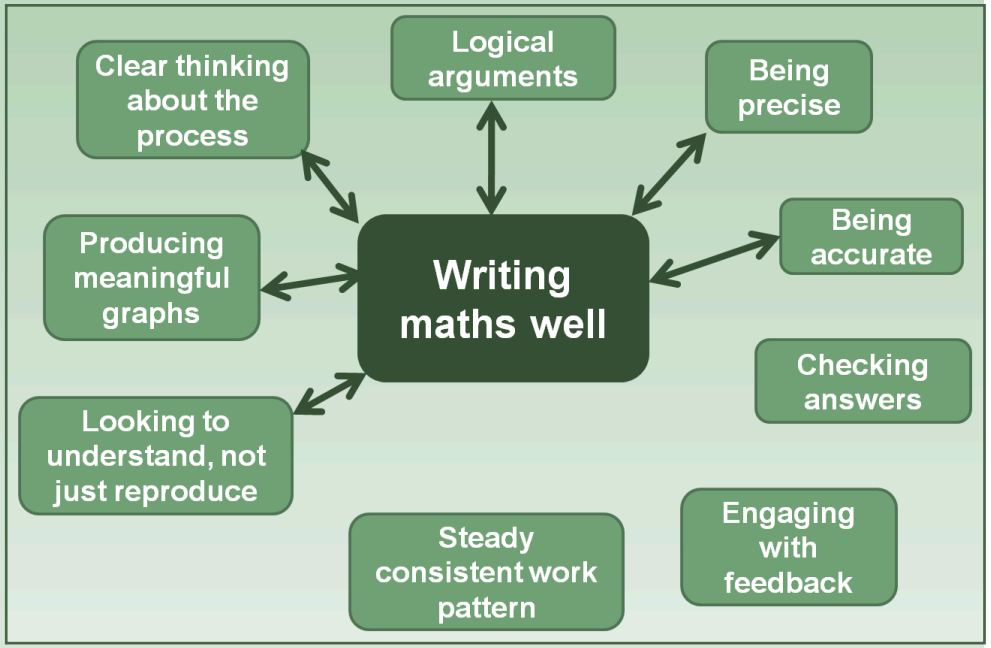
What is 'good writing'? Whose benefit?

I want them to write well in all their work, not just what they hand in.



What is 'good writing'? Whose benefit?

I want them to write well in all their work, not just what they hand in.



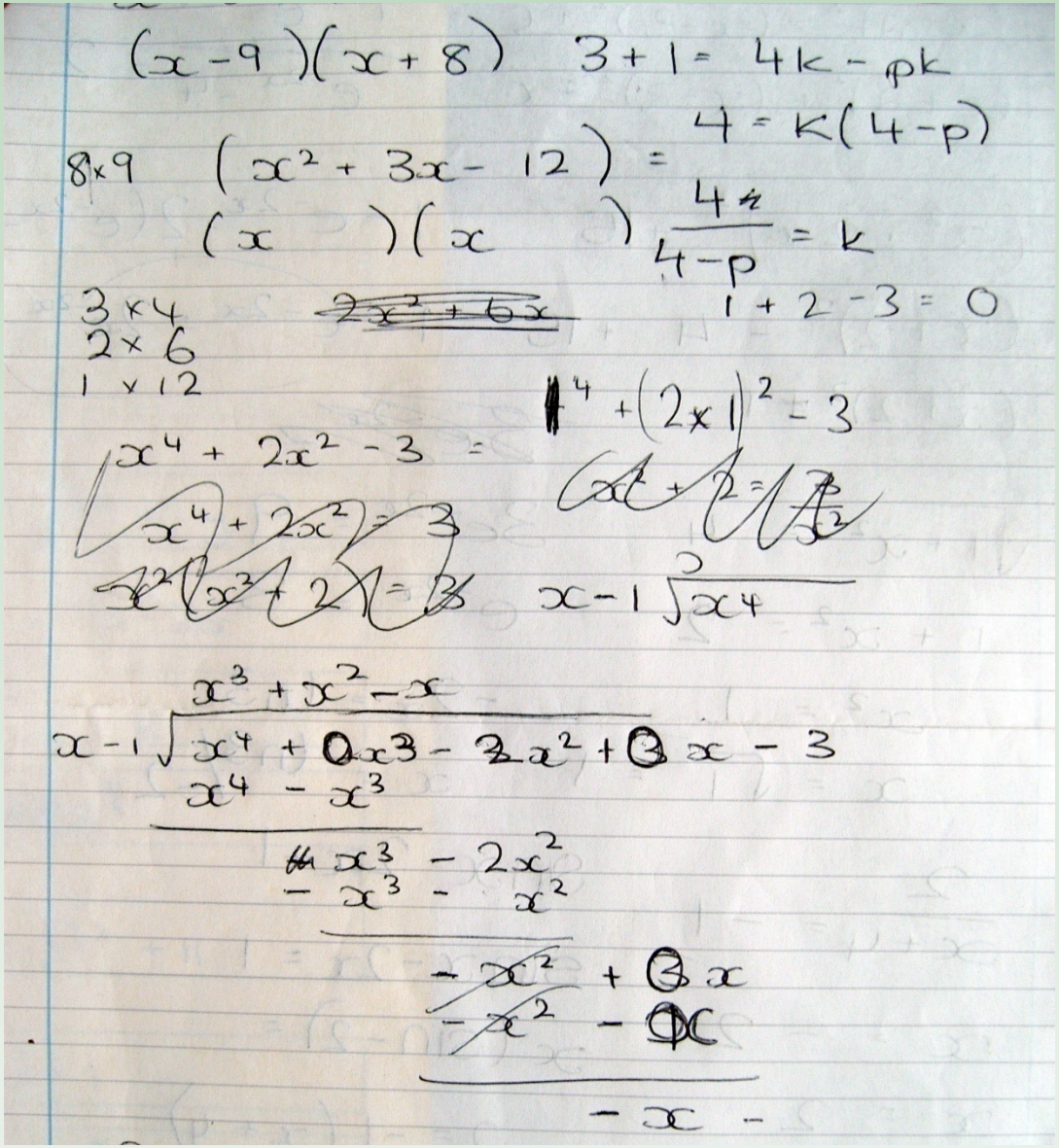
Differences between
- private work?
- exams?
- "Find the solution of…"?
- "Write a report on…"?

- Sentences? Pseudo-sentences?
- Abbreviations?

"Sub (1) → (2):"

"Substituting equation (1) into equation (2) gives"

What is 'good writing'? Whose benefit?



What is 'good writing'? Whose benefit?

A5 Prove the identity $\cosh^{-1} x \equiv \ln (x \pm \sqrt{x^2 - 1})$.

$y = \cosh^{-1} x$

$x = \cosh y = \frac{e^y + e^{-y}}{2}$ (× 2)

$2x = e^y + e^{-y}$

$(\times e^y)$ $2xe^y = e^y e^y + e^{-y} e^y$ (1)

$(e^y)^2 - 2xe^y + 1$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$e^y = \frac{2x \pm \sqrt{2x^2 - 4}}{2}$

$= x \pm \sqrt{x^2 - 1}$

$y = \ln x \pm \sqrt{x^2 - 1}$ QED

What is 'good writing'? Whose benefit?

B8 (c) (i) Let $y = \cosh^{-1} x$ (1)

$\Rightarrow \cosh y = x$

Using defn of cosh

$$x = \frac{e^y + e^{-y}}{2}$$

$$\Rightarrow 2x = e^y + e^{-y}$$

$$\Rightarrow 0 = (e^y)^2 - 2xe^y + 1$$

Using the quadratic formula

$$e^y = \frac{2x \pm \sqrt{4x^2 - 4}}{2}$$

$$= x \pm \sqrt{x^2 - 1}$$

$$\Rightarrow y = \ln\left(x \pm \sqrt{x^2 - 1}\right)$$

From (1) $\cosh^{-1} x = \ln\left(x \pm \sqrt{x^2 - 1}\right)$ as required.

What is 'good writing'? Whose benefit?

To find the solution of

$$\ddot{y} + 12\dot{y} - \frac{6y}{125} = -\frac{49}{5} \qquad (1)$$

we first write down the auxiliary equation

$$\lambda^2 + 12\lambda - \frac{6}{125} = 0.$$

Solving this gives

$$\lambda_1 = -6 - \frac{\sqrt{22530}}{25} \quad \text{and} \quad \lambda_2 = -6 + \frac{\sqrt{22530}}{25}.$$

Since the roots are real and distinct, we know that the complimentary function y_{CF} is of the form

$$y_{CF} = Ae^{\left(-6-\frac{\sqrt{22530}}{25}\right)t} + Be^{\left(-6+\frac{\sqrt{22530}}{25}\right)t}.$$

Considering the inhomogeneity in equation (1) we expect the particular integral to be of the form $y_{PI} = C$ where C is a constant, and so

$$\dot{y}_{PI} = \ddot{y}_{PI} = 0.$$

Substituting these into (1) we obtain

$$-\frac{6C}{125} = -\frac{49}{5}$$

and hence

$$y_{PI} = C = \frac{1225}{6}.$$

The general solution is given by $y = y_{CF} + y_{PI}$ and hence we can say that

$$y = Ae^{\left(-6-\frac{\sqrt{22530}}{25}\right)t} + Be^{\left(-6+\frac{\sqrt{22530}}{25}\right)t} + \frac{1225}{6}.$$

Reflected in: *Course design/structure*

Old

- Solving, transposing
- Identities
- Quadratics
- Logs & powers
- Exponential functions
- Differentiation
    -
    -
    -
- Complex numbers
- Taylor and Maclaurin Series
- Introduction to ODEs

Reflected in: *Course design/structure*

Old

- Solving, transposing
- Identities
- Quadratics
- Logs & powers
- Exponential functions
- Differentiation
  - •
  - •
  - •
- Complex numbers
- Taylor and Maclaurin Series
- Introduction to ODEs

New

- Clear/legible/neat
- What do we mean by ‘statement’/expression/ equation…?
- Connecting statements
- Using = correctly
- Checking answers
  - •
  - •
  - •
- Using ‘implies that’
- Logical arguments
- Proof

Reflected in: *Handouts and lectures*

- Everything written 'properly'
  - solutions/handouts (easy)
  - lectures (harder)

Reflected in: *Handouts and lectures*

- Everything written 'properly'
  - solutions/handouts (easy)
  - lectures (harder)

- Commentary about the writing
  - lectures (takes some getting used to)
  - handouts (sometimes tricky)

Reflected in: *Coursework and mark scheme*

- Tiny weekly assignment
  - (relatively) quick to mark
  - frequent rapid feedback.

- Graded twice (A-D, a-d)
  - **equal weight**
  - once for method/solution
  - once for 'being a good mathematician'

    Cumulative, up to this week's 'good maths' theme, which brings me to…

Extremely gradual development

- **Week 1**
  - Clear/neat/legible
  - Flows from left to right, top to bottom
  - 7, z, 2, s, S, 5, x, X, c, C, ¼x, p, ρ, d, δ...
  - Summarise your result

Extremely gradual development

- **Week 1**
  - Clear/neat/legible
  - Flows from left to right, top to bottom
  - 7, z, 2, s, S, 5, x, X, c, C, ¼x, p, ρ, d, δ...
  - Summarise your result
- **Week 2**
  - Statements/equations/formulae/expressions
  - Explain where your statements are from
  - A little about proving/disproving identities

Extremely gradual development

- **Week 1**
  - Clear/neat/legible
  - Flows from left to right, top to bottom
  - 7, z, 2, s, S, 5, x, X, c, C, ¼x, p, ρ, d, δ...
  - Summarise your result
- **Week 2**
  - Statements/equations/formulae/expressions
  - Explain where your statements are from
  - A little about proving/disproving identities
- **Week 3**
  - Use and misuse of the equals sign…

Difficulties

- Redesign/repackaging material (ongoing)
- Student resistance?
- Staff resistance?
- Staff undermining?
  - 'doing the maths'
- Feedback is time-consuming especially at first
  - pays dividends later?

Perceived advantages for students

- Clarity about
  - what is really important
  - that this module is not ‘just revision’
  - that we expect them to work differently from school
- Explicitly teaching and rewarding the skills we value
- Regular frequent feedback
  - gradual development
  - encourages habit of constant work
- Embedding key mathematical skills with content
- **They deal with the 'sums' more successfully**

Does it work?

Quantitative (with a huge pinch of salt)

- 2008-9 exam mean: 52%, pass rate: 64%
- 2009-10 exam mean: 54%, pass rate: 81%

(Exam marking based *primarily* around 'content' rather than 'good maths')

Qualitative

- I see better written maths and less nonsense
- Most students appreciating the importance of key skills and engaging with improving them
- Hoping good habits last a (university) lifetime
  - current 2nd years show improvement on previous cohort??

Student reaction

“It was nice that the first lecture was all centred around how we as a group could become 'good mathematicians'”

“Really enjoying this module as we have to start thinking about how we are actually communicating our answers”

“I find this module quite interesting, I think the idea is to get people to think about mathematics in a more logical way.”

“I am finding it a little bit awkward and difficult in methods because of writing our maths out in sentences. I am finding it highly time consuming and difficult to get used to. However I do see how useful it will be to do this”

Student reaction

“Some of the techniques that have been asked of us to become better mathematicians I have started to really pick up and just do them naturally.”

“Although the content isn't new, some of the thinking behind it is, and it is allowing me to get a more solid understanding to this work and other things I have done in the past.”

Summary

Changes involved

- identifying the skills/attitudes/habits we want our students to develop
  - maths is more than just 'content'.
- making these explicit (to tutor and students) and
  - giving them time
  - giving them credit
  - not begrudging them.

Results

- Students still learn the content, but deal with it more effectively?