Teaching Students To Write Mathematics

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Outline

- The problem.

Outline

- The problem.
- My solution(s).

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- The problem.
- My solution(s).
- The way forward.

The problem

Example

Year 1 question: Is $\{1, 2, 3, 4, 5, 6, 7\}$ under multiplication $\mod 8$ a group?

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Student answer:

The problem

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Year 1 question: Is $\{1, 2, 3, 4, 5, 6, 7\}$ under multiplication $\bmod 8$ a group?

Student answer: not closed.

The problem

Example

Year 1 question: Is $\{1, 2, 3, 4, 5, 6, 7\}$ under multiplication $\bmod 8$ a group?

Student answer: not closed. set contains 0.

Writing mathematics

- Forces thinking.

Writing mathematics

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- Fantastic skill to have.

Writing mathematics

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- Fantastic skill to have.
- Marking is a joy.

Myth

Myth:

Myth

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Learning mathematics teaches

Myth

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Learning mathematics teaches

- critical thinking and

Myth

Myth:

Learning mathematics teaches

- critical thinking and
- logical thinking

Myth 2

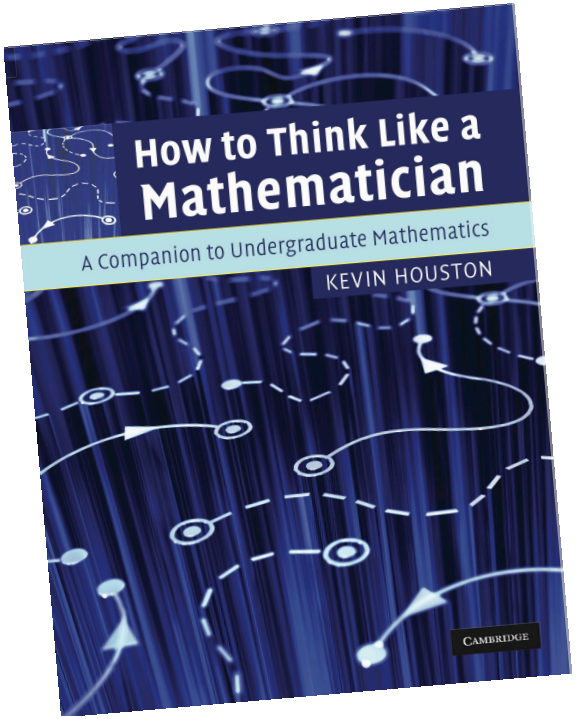
‘Tell students to write their work so that someone else can understand it’.

Myth 2

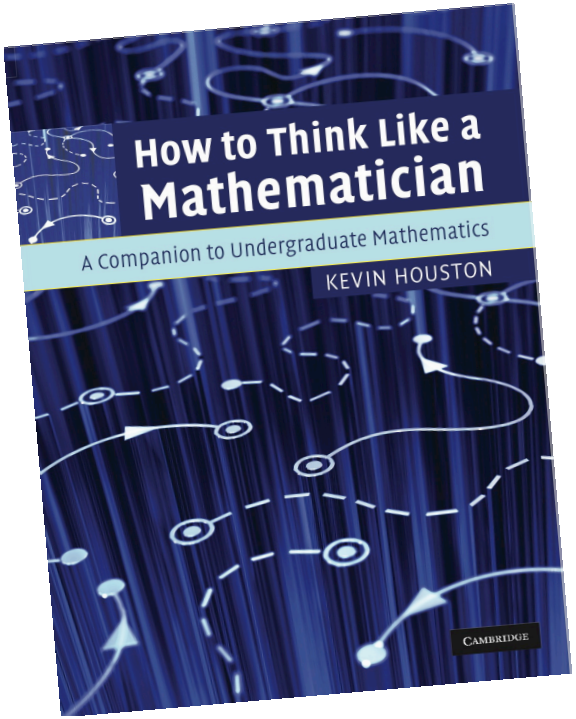
‘Tell students to write their work so that someone else can understand it’.

If only it were that simple.

My approach

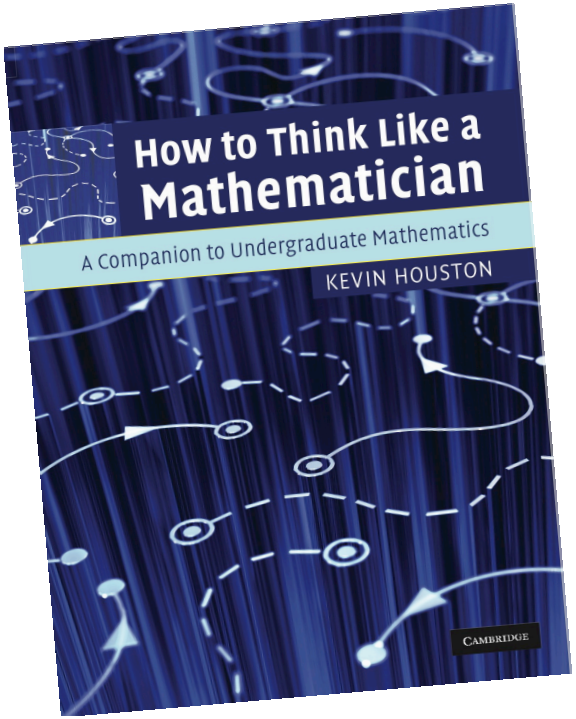

How to Think Like a Mathematician
A Companion to Undergraduate Mathematics
KEVIN HOUSTON
CAMBRIDGE

My approach



- Teach thinking.

My approach



- Teach thinking.
- Teach writing.

Thinking and writing

To write clearly, students need to think clearly.

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To think clearly, students need to write clearly.

Reasons

- A good university education prepares a student for life.

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Reasons

- A good university education prepares a student for life.
- Writing well is an extremely useful skill.
- Students choose mathematics as they do not like writing essays.
- Low hanging fruit: Small effort gives major boost for student.

How to write mathematics booklet.

How to write mathematics

How to write mathematics booklet.

Chapters 3 and 4 of my book.

Copies of samples on my website.

www.kevinhouston.net

What the students are told

- Write in sentences.

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- Use punctuation.

What the students are told

- Write in sentences. This advice has precedence over all others. This can really change the way you present your work.
- Use punctuation. That means a capital letter at the start of sentences and a full stop at the end. Doing this makes your work so much clearer.

What the students are told

- Use grammar correctly.

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- Common error:

What the students are told

- Use grammar correctly.
- Common error: ‘Mathematics is highly symbolic so if I just provide some mathematical symbols I’m doing maths.’

What the students are told

- Use grammar correctly.
- Common error: ‘Mathematics is highly symbolic so if I just provide some mathematical symbols I’m doing maths.’ This is wrong. Symbols are merely shorthand for certain concepts; they need to be incorporated into sentences for there to be any meaning.

What the students are told

Others:

- Readers are not psychic. Explain what you are doing.

What the students are told

Others:

- Readers are not psychic. Explain what you are doing.
- Explain your assertions

What the students are told

Others:

- Readers are not psychic. Explain what you are doing.
- Explain your assertions
- Use of symbols

What the students are told

Others:

- Readers are not psychic. Explain what you are doing.
- Explain your assertions
- Use of symbols
- Expressing yourself clearly

My favourite

Curse of the implication symbol.

My favourite

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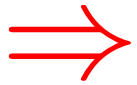
I can understand why students use the implication symbol:



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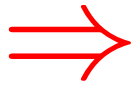

It makes a proof or the answer to an exercise look mathematical.

Another example

Find, in the form $y = mx + c$, the invariant lines of the transformation with matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

Solution

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ mx+c \end{bmatrix} = \begin{bmatrix} mx+c \\ x \end{bmatrix}$$

$$\Rightarrow x = m(mx+c)+c$$

$$\Rightarrow (1-m^2)x = (m+1)c, \quad \text{for all } x.$$

Put $x = 0$, then $(m+1)c = 0 \quad \Rightarrow m = -1$ or $c = 0$.

Then $(1-m^2)x = 0$, for all $x \quad \Rightarrow m = \pm 1$.

So, $\quad m = -1 \quad$ or $\quad m = 1, c = 0$

$\Rightarrow y = -x + c \quad$ or $\quad y = x$.

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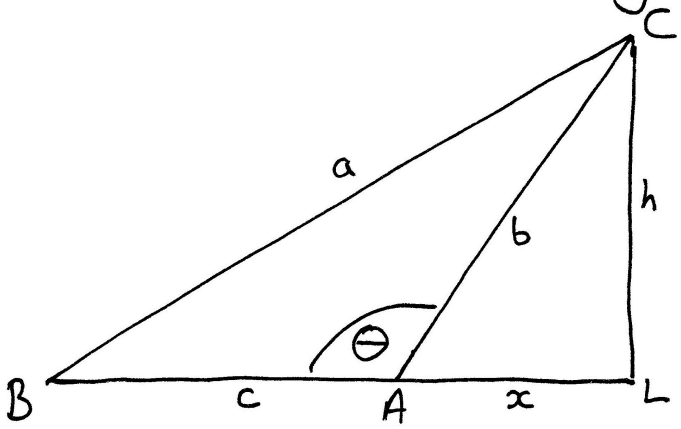
Worked example

Geometry Examples 1



Θ = obtuse angle

$\triangle CBL$

$a^2 = (c+x)^2 + h^2$

$a^2 = c^2 + 2cx + h^2 + x^2$

$h^2 + x^2 = b^2$

$\triangle CLA$

$b^2 = h^2 + x^2$

$a^2 = b^2 + c^2 + 2cx$

In $\triangle CLA$ $\quad \frac{x}{b} = \cos(180 - \theta)$

$\Rightarrow x = -b\cos\theta$ Sub in to

$a^2 = b^2 + c^2 - 2cb\cos\theta$

Worked example

In the booklet students are taken through the errors in the example and shown how to produce a better solution.

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Videos of the example are available: www.youtube.com/user/DrKevinHouston

Weekly exercises

How is this applied?

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- Intake 148 Home/EU and 20 International.

Weekly exercises

How is this applied?

- Intake 148 Home/EU and 20 International.
- Current offer AAB, A in Maths, half have Further Maths A or AS level.

Weekly exercises

How is this applied?

- Intake 148 Home/EU and 20 International.
- Current offer AAB, A in Maths, half have Further Maths A or AS level.
- Six tutees.

Weekly exercises

How is this applied?

- Intake 148 Home/EU and 20 International.
- Current offer AAB, A in Maths, half have Further Maths A or AS level.
- Six tutees.
- Weekly exercises.

Weekly exercises

- No tolerance of errors?

Weekly exercises

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- Plenty of feedback. HTWLAM page number.

Weekly exercises

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- Do they read it?

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- Do they read it?
- Student resistance.

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- Students dispirited.

Weekly exercises

- No tolerance of errors?
- Plenty of feedback. HTWLAM page number.
- Too much feedback?
- Do they read it?
- Student resistance.
- Students dispirited.
- Changes to our first year. Groups of 12.

Conclusion

Conclusion

They hate it

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They hate it
at first

Conclusion

They hate it
at first
but are grateful at the end.

Conclusion

They hate it
at first
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Not very scientific.

Conclusion

- Colleagues (broadly) in favour.

Conclusion

- Colleagues (broadly) in favour.
- Can undermine.

Conclusion

- Colleagues (broadly) in favour.
- Can undermine.
- Resistance to ‘skills’.

The way forward

- Low hanging fruit. Big difference. In practice and in research.

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- Chapters 3 and 4 are free on my websites.
  - Leeds School of Maths.
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The way forward

- Low hanging fruit. Big difference. In practice and in research.
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- Contribute examples of good practice. Make Latex exercises available.

Thanks

Thanks for listening.